Minisymposium 11 - Geometrische Analysis

Chord-Arc Submanifolds of Arbitrary Codimension

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A rectifiable Jordan curve Γ that goes through ∞ is called a *chord-arc curve with constant* κ, iff

$$|s-t| \leq (1+\kappa)|z(s)-z(t)|$$

for all $s, t \in \mathbb{R}$, where $z(.)$ denotes an arc length parametrization of Γ.

In [1,2,3] S. Semmes introduces several different analogs to the chord-arc constant for hypersurfaces of Euclidean spaces. Among other things he proves that each of these constants is small, if one of them is small, and moreover he shows that surfaces with a small chord-arc constant are homeomorphic to a hyperplane.

We extend the notion of chord-arc surfaces and constants to submanifolds of arbitrary codimension. Following ideas of Semmes one can show that they contain big pieces of Lipschitz graphs, if the chord-arc constant is sufficiently small. Using this and a smoothing argument, we are able to show that n-dimensional submanifolds with small chord-arc constants are not only homeomorphic to Euclidean n-space, but even unknotted.

References

[1] Stephen Semmes. Chord-Arc with small constant. I. *Adv. Math.*, 85(2), 198-223, 1991

[2] Stephen Semmes. Chord-Arc with small constant. II. Good parametrizations. *Adv.Math.*, 88(2), 170-199, 1991

[3] Stephen Semmes. Hypersurfaces in $\mathbb{R}^n$ whose unit normal has small BMO norm. *Proc. Amer. Math. Soc.*, 112(2), 403-412, 1991