Minisymposium 19 - Random Discrete Structures and Algorithms

On the bandwidth conjecture of Bollobás and Komlós

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The study of sufficient degree conditions, on a given graph G, which imply that G contains a particular spanning subgraph H is one of the central areas in graph theory. A well known example of such a result is Dirac's theorem. It asserts that any graph G on n vertices with minimum degree at least $n/2$ contains a spanning, so called Hamiltonian, cycle.

In my talk we discuss related results for 3-chromatic graphs H of bounded maximum degree and small bandwidth. In particular we show that: *For every Δ and $\gamma > 0$ there exist a constant $\beta > 0$ such that for sufficiently large n the following holds. If G is an n-vertex graph with minimum degree $\delta(G) \geq (2/3 + \gamma)n$, then it contains a spanning copy of every 3-chromatic n-vertex graph H with maximum degree $\Delta(H) \leq \Delta$ and bandwidth $\mathrm{bw}(H)$ at most βn, where $\mathrm{bw}(H) = \min_\sigma \max_{uv \in E(H)} |\sigma(u) - \sigma(v)|$ with the minimum ranging over all bijections from $V(H)$ to $[n]$.* This settles a conjecture of Bollobás and Komlós for the special case of 3-chromatic graphs H. It is known that the minimum degree condition on G is asymptotically best possible.

The proof is based on Szemerédi's regularity lemma and the so called blow-up lemma. This is joint work with Julia Böttcher and Anusch Taraz from TU München.