PHD SEMINAR ON GALOIS EXTENSIONS OF SYMMETRIC RING SPECTRA

Classical Galois theory states that there is a one-to-one correspondence between intermediate fields of a field extension L/K satisfying certain properties, usually summarized under the name *Galois extension*, and subgroups of the Galois group of the extension consisting of the automorphisms of L leaving the subfield K fixed. The aim of the seminar is to understand how to generalize this theory to the context of ring spectra. It is immediately clear that in order to do algebraic construction one requires a category of highly structured ring spectra, *i.e.* one having a closed symmetric monoidal smash product. Since the category of symmetric spectra is probably the most simple such category, we would suggest to work with these. To make things easier we first consider the problem of generalizing to Galois theory of commutative rings which is an interesting subject in its own right. The strong formal similarities between commutative rings and commutative ring spectra enable us to generalize further pretty easily. The seminar should cover enough theory to be able to prove the main theorem of Galois theory (at least sketch it) in the context of commutative symmetric ring spectra and give interesting examples of such Galois extensions.

The seminar is based on the paper [Rog05] which uses S-modules instead of symmetric spectra but is very easily adapted to symmetric spectra. We further suggests to work with symmetric spectra based on topological spaces rather than simplicial sets. The organization naturally falls into four parts:

- **Galois theory of finite field extensions.** The task here is to grab his or her favorite algebra textbook and remind us all of what we might have forgotten since we are PhD students. It is desirable to present things more efficiently and abstractly than one would expect in an undergraduate lecture, *e.g.* as it is done in the first two chapters of [BJ01]. (probably two talks).
- **Galois theory of commutative rings**. These talks are based on the first chapter of the book [Gre92]. According to the interest we should try to give complete proofs. The minimal requirement is to first define Galois extensions and show that they recover the classical notion when specializing to fields, and second to prove the Galois correspondence theorem in the commutative ring context. Another reference is [Mag74] (two or three talks)
- **Symmetric spectra, ring spectra and module spectra.** Before we can start with Galois theory of ring spectra it is advisable to recall the technical frame work. First of all we should recall model categories and give at least the examples of topological spaces and spectra of Bousfield Friedlander type (or naive prespectra as some call them). Then define symmetric spectra (based on topological spaces) focusing on the closed symmetric monoidal smash product. A discussion of the stable model structure is relevant and the lifting of this structure to categories of module, ring and algebra spectra. Maybe it suffices to discuss the ideas and main features of the model

structures rather than proving their existence. References include [HSS00], [MMSS01], [Shi04] and [SS00]. (about three talks)

- **Galois extensions of symmetric ring spectra.** Here one must define what a Galois extension of symmetric ring spectra is and give some examples. In [Rog05] one works in the E-local category,but for the sake of simplicity we should work globally,*i.e.* S-locally, even if there are extremely interesting examples in local cases. For some examples one needs to do calculations using various spectral sequences (*e.g.* the homotopy fixed points spectral sequence, Künneth and Tor spectral sequence) so we might want to have an extra talk on theses. To be able to proof the main Galois correspondence theorem we need to compute mapping spaces for which Goerss-Hopkins theory is required. This will also be discussed in the research seminar, so we could have a short talk adapting to our needs. References are [Rog05], [EKMM97], [GH04b] and [GH04a]. The book [BR04] is also useful. (three or four talks)

Talks

(1) **Galois extensions of fields**, (Hamzeh Zarghani, Saeid)
(2) **Galois extensions of commutative rings I**, (Wang, Juan)
(3) **Galois extensions of commutative rings II**, (Müllner, Daniel)
(4) **Symmetric spectra**, (Ullmann, Mark)
(5a) **Symmetric ring and module spectra**, (Möllers, Jan)
(5b) **Bousfield localization**, (Möllers, Jan)
(6) **Dualizable spectra**, (Langer, Martin)
(7) **Some spetral sequences in stable homotopy thoery**, (Deniz, Ferit)
(8) **Galois theory for symmetric ring spectra I: Definitons and Examples**, (Groth, Moritz)
(9) **Example: topological K-theory, $KO \to KU$**, (von Bodecker, Hanno ?)
(10) **Galois theory for symmetric ring spectra II: base change and seperabel extenions**, (Recker, Holger)
(11) **Galois theory for symmetric ring spectra III: Galois correspondens for ring specra**, (Weiner, Arne)

References

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[BR04] Andrew Baker and Birgit Richter, editors. *Structured ring spectra*, volume 315 of *London Mathematical Society Lecture Note Series*. Cambridge University Press, Cambridge, 2004.

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[GH04a] P. G. Goerss and M. J. Hopkins. Moduli problems for structured ring spectra. `http://hopf.math.purdue.edu//Goerss-Hopkins/obstruct.pdf`, 2004. Preprint.

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[HSS00] Mark Hovey, Brooke Shipley, and Jeff Smith. Symmetric spectra. *J. Amer. Math. Soc.*, 13(1):149–208, 2000.

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