Torus Knots*

Torus knots are quite popular space curves because they represent the simplest way to write down knotted curves in $\mathbb{R}^3$. Our knots are parametrized as

$$c(t) := \begin{pmatrix} aa + bb \cdot \cos(dd \cdot t) \cdot \cos(ee \cdot t) \\ aa + bb \cdot \cos(dd \cdot t) \cdot \sin(ee \cdot t) \\ cc \cdot \sin(dd \cdot t) \end{pmatrix}$$

with defaults $aa = 3,\ bb = 1.5,\ cc = 1.5,\ \ dd = 5,\ ee = 2.$

The default `Morph` changes the torus size. If, before the morph, one chooses in the Action Menu `Show As Tube` and `Parallel Frame` then one notices that the twisting of the tube (see the ATO of the helix for more details) is clearly visible already for rather small changes of the shape of the torus.

The Action Menu has also the entry `Show Dotted Torus`. Selecting it adds the torus to the picture. This is more spectacular when viewing in `Anaglyph Stereo Vision`, through red/green filter glasses. Observe that our brain gets these several thousand dots sorted out into corresponding pairs of red and green dots that then form the torus surface in $\mathbb{R}^3$ - and this seems to happen instantly.

The best method to get a feeling for the curvature of a space curve is to select in the Action Menu `Show Osculating Circles` & `Evolute`. The Radius r of the circle

* This file is from the 3D-XplorMath project. Please see: http://3D-XplorMath.org/

is the *radius of curvature* of the curve at the current point and $\kappa := 1/r$ is called the *curvature* (at that point). The direction from $c(t)$ to the midpoint of the osculating circle determines always the direction of the second basis vector of the Frenet frame.
If one uses the Parallel Frame, then one has to represent the curvature by a vector of length κ in the plane spanned by the two normal vectors of the Parallel frame. If one has selected, in the Action Menu, `Parallel Frame` and clicks `Show Repére Mobile` then this curvature vector is drawn, together with its past history, in each normal plane. – The last entry in the Action Menu, `Show Frenet Integration` does the opposite: if the curvature vector function is given in the initial normal plane then the demo reconstructs the curve by integrating the Frenet equation.

H.K.