

	Morse theory	Hamiltonian Floer theory
Space	M closed manifold	$\widetilde{\mathcal{L}}_0 M$ for $M = (M, \omega)$ compact symplectic
Function(al)	$f : M \rightarrow \mathbb{R}$	$\mathcal{A}([\gamma, u]) = - \int_{D^2} u^* \omega + \int_0^1 H_t(\gamma(t))$
Critical points	$x \in M$ such that $df(x) = 0$	$\gamma : S^1 \rightarrow M$ such that $\dot{\gamma}(t) = X_t^H(\gamma(t))$
Non-degeneracy condition	$ \partial_i \partial_j f(x) \neq 0$ (i.e. f is Morse)	$ id - d\phi_1^H \neq 0$ (i.e. H is non-degenerate)
Index	Morse index	Conley–Zehnder index ¹
Auxiliary metric	g a Riemannian metric on M	$\langle -, - \rangle_J := \int_0^1 \omega(-, J-)$ for compatible J , which defines a Riemannian metric on $\widetilde{\mathcal{L}}_0 M$
Negative gradient equation	$\dot{\gamma}(t) + \nabla^g f(\gamma(t)) = 0$	$\bar{\partial}_{H,J}(u) := \partial_s u + J(u)(\partial_t u - X_t^H(u)) = 0$
Transversality condition	(f, g) is Morse–Smale	(H, J) is regular
Compactness	Morse compactness: <i>a sequence of negative gradient trajectories limits (up to subsequence) to a broken gradient trajectory</i>	Gromov–Floer compactness: <i>in the absence of sphere bubbles, a sequence of Floer trajectories limits to a broken Floer trajectory</i> ²
Coefficients	$\mathbb{Z}/2$ in general; can work over an arbitrary commutative ring if the moduli spaces admit coherent R -orientations	if $\omega \cdot \pi_2(M) = 0$, then can work over $\mathbb{Z}/2$, and over \mathbb{Z} after choosing coherent orientations. In general, must work over a Novikov ring Λ_R . ³

¹Mod 2 unless we assume that $c_1(TM) \cdot \pi_2(M) = 0$

²As we briefly discussed in class, Floer cohomology can still be defined in the presence of sphere bubbles, but this requires much more sophisticated methods.

³Typically, one must assume that R contains \mathbb{Q} , which also requires a choice of coherent orientations. This can be avoided under some assumptions, such as M being Calabi–Yau.