SAMPLE PROBLEMS FOR QUIZ 2

Problem 1. Find the Galois group of the splitting field of the polynomial $t^3 - 10$ over the fields $\mathbb{Q}$, $\mathbb{Q}(\sqrt{2})$, $\mathbb{Q}(\sqrt{-3})$, $\mathbb{F}_{13}$.

Problem 2. Let K be the splitting field of $t^5 - 2$ over $\mathbb{Q}$, $M = \mathbb{Q}(\alpha) \subset K$ where $\alpha = \sqrt[5]{2}$ and $G := Gal(K/\mathbb{Q})$. Show that:

(a) K contains a subfield $L = \mathbb{Q}(\zeta)$ where $\zeta^5 = 1, \zeta \neq 1$,
(b) $[L : \mathbb{Q}] = 4$, $[K : L] = 5$ and $Gal(K/L) = \mathbb{Z}/5\mathbb{Z}$,
(c) There exists unique $\tau \in Gal(K/L)$ such that $\tau(\alpha) = \zeta\alpha$ and $\tau^5 = e$.
(d) There exists $\sigma \in Gal(K/M)$ such that $\sigma(\zeta) = \zeta^2$, and $\sigma^4 = e$.
(e) $\sigma\tau\sigma^{-1} = \tau^2$,
(f) Any element $g \in G$ can be written uniquely in the form $g = \sigma^a\tau^b, a \in \mathbb{Z}/4\mathbb{Z}, b \in \mathbb{Z}/5\mathbb{Z}$.

Problem 3. Find the Galois group of the splitting field of the polynomial $t^4 - x$ over the field $\mathbb{R}(x)$.

Problem 4. Find the Galois group of the splitting field of the polynomials $t^3 + t + x$, $t^3 + x^2t + x^3$ over the field $\mathbb{C}(x)$.

Problem 5. Show that the extension $\mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5}) : \mathbb{Q}$ is normal and find $Gal(\mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5})/\mathbb{Q})$.